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Small steps towards solving the bosonic master-field equation of the IIB matrix model

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1. Algebraic equation

The equation of the title is an **algebraic equation** for D traceless Hermitian matrices $\widehat{a}^{\,\mu}$ of dimension $N \times N$:

$$
\begin{aligned}
i\,\big(\widehat{p}_k - \widehat{p}_l\big)\,\widehat{a}^{\,\mu}_{\,kl} &= \Big[\widehat{a}^{\,\nu},\,\big[\widehat{a}^{\,\nu},\,\widehat{a}^{\,\mu}\big]\Big]_{kl} - F\,\frac{1}{\mathcal{P}(\widehat{a})}\,\frac{\partial\,\mathcal{P}(\widehat{a})}{\partial\,\widehat{a}^{\,\mu}_{\,lk}} + \widehat{\eta}^{\,\mu}_{\,kl}\,, && \text{(1a)}\\
\mathcal{P}(\widehat{a}) &= \text{homogeneous polynomial of degree } K\,, && \text{(1b)}\\
K &\equiv \big(D-2\big)\,\big(N^2-1\big)\,, && \text{(1c)}\\
(D,\,F) &= (10,\,1), \quad N \gg 1\,, && \text{(1d)}
\end{aligned}
$$

with matrix indices k, l running over $\{1,\,\ldots\,,\,N\}$ and directional indices μ, ν running over $\{1,\,\ldots\,,\,D\}$, while ν in (1a) is implicitly summed over.
The $\widehat{p}_k$ are fixed uniform random numbers and the $\widehat{\eta}^{\,\mu}_{\,kl}$ fixed Gaussian random numbers.
There is an explicit expression for the Pfaffian $\mathcal{P}$ to be discussed later.

1. Algebraic equation

The algebraic equation (1) is quite a challenge for mathematics and computational science.

But why is that equation also of interest to physics?

Well, the answer is that its solution may contain information about

$\boxed{\text{the emergence of spacetime and the birth of the Universe}}$.

Here, I can only give some background in an ultrashort version (a short version appears in Appendix A), as the main focus will be on discussing preliminary solutions of this algebraic equation.

2. Background (ultrashort version)

We start from the IIB matrix model [1, 2], which reproduces the structure of the light-cone string field theory of type-IIB superstrings.

The IIB matrix model has a **finite number** of $N \times N$ traceless Hermitian matrices: ten bosonic matrices A^μ and eight fermionic (Majorana–Weyl) matrices Ψ_α.

The partition function Z of the IIB matrix model is defined by a “path” integral over A and Ψ with a weight factor $\exp[-S_{\text{bos}}(A) - S_{\text{ferm}}(\Psi,\, A)]$. The fermionic matrices Ψ can be integrated out exactly (Gaussian integrals) and then give the Pfaffian $\mathcal{P}(A)$.

For strings of bosonic observables, the expectation values are defined by the same A-integral as Z, that is, involving the exponential weight factor with the bosonic action, $\exp[-S_{\text{bos}}(A)]$, and the Pfaffian $\mathcal{P}(A)$.

For large N, these expectation values can also be obtained by inserting the matrices $\widehat{A}^{\,\mu}$ of the so-called **master-field** [3, 4] directly into the observables, without need of any integration.

2. Background (ultrashort version)

Recently, we have suggested [5] that precisely the master-field matrices $\widehat{A}^{\mu}$ of the IIB matrix model may give rise to an emergent classical spacetime.

Assuming that the matrices $\widehat{A}^{\mu}$ of the IIB-matrix-model master field are known and that they are approximately band-diagonal, it relatively easy [5] to extract a discrete set of spacetime points $\{\widehat{x}_k^{\mu}\}$ and an interpolating (inverse) metric $g^{\mu\nu}(x)$.

It has also been established that, in principle, it is possible to get, from appropriate distributions of the extracted spacetime points $\{\widehat{x}_k^{\mu}\}$, the metrics of the Minkowski and the spatially flat Robertson–Walker spacetimes. See the recent review [6] for further discussion.

But, instead of <u>assuming</u> the matrices $\widehat{A}^{\mu}$, we want to **calculate** them. And, for that, we need an equation . . .

3. Equation and solutions

GOOD NEWS:

the master-field equation has already been established, nearly 40 years ago, by Greensite and Halpern [4], who write in the first line of their abstract:

“We derive an exact algebraic (master) equation for the euclidean master field of any large-N matrix theory, including quantum chromodynamics.”

Now, “any” means “any” and we may as well consider the large-N IIB matrix theory of Kawai and collaborators [1, 2].

3.1 Bosonic master-field equation

Building on this work by Greensite and Halpern [4], we then have the IIB-matrix-model bosonic master field in "quenched" form [5]:

$$\widehat{A}^{\,\mu}_{\,kl} = e^{i\,(\widehat{p}_k - \widehat{p}_l)\,\tau_{\text{eq}}}\;\widehat{a}^{\,\mu}_{\,kl}\,. \tag{2a}$$

The dimensionless time τ_{eq} in (2a) must have a sufficiently large value in order to represent an equilibrium situation (τ is the fictitious Langevin time of the stochastic-quantization procedure).
The τ-independent matrix $\widehat{a}^{\,\mu}$ on the right-hand side of (2a) solves the following algebraic equation [5]:

$$i\,(\widehat{p}_k - \widehat{p}_l)\;\widehat{a}^{\,\mu}_{\,kl} \;=\; -\frac{\partial\, S_{\text{eff}}}{\partial\, \widehat{a}_{\mu\; lk}} + \widehat{\eta}^{\,\mu}_{\,kl}\,, \tag{2b}$$

in terms of the master momenta $\widehat{p}_k$ (uniform random numbers) and the master noise matrices $\widehat{\eta}^{\,\mu}_{\,kl}$ (Gaussian random numbers). The algebraic equation (2b) is, of course, precisely (1).

3.2 Solutions of the simplified equation

The algebraic equation (1) is rather formidable and it makes sense to first consider the simplified equation obtained by setting $F = 0$:

$$i\left(\widehat{p}_k - \widehat{p}_l\right)\widehat{a}^{\,\mu}_{\;kl} \quad = \quad \Big[\widehat{a}^{\,\nu},\, \big[\widehat{a}^{\,\nu},\, \widehat{a}^{\,\mu}\big]\Big]_{kl} + \widehat{\eta}^{\,\mu}_{\;kl}\,. \tag{3}$$

The matrices $\widehat{a}^{\,\mu}$ are $N \times N$ traceless Hermitian matrices and the number of variables is

$$N_{\text{var}} = D\left(N^2 - 1\right), \tag{4}$$

which grows rapidly with increasing N. Remark also that the simplified equation (3) is essentially a **cubic polynomial**.

It appears impossible to obtain a general analytic solution of (3) in terms of the master constants $\widehat{p}_k$ and $\widehat{\eta}^{\,\mu}_{\;kl}$. Instead, we will try to get solutions for an explicit choice for the random master constants.

3.2 Solutions of the simplified equation

For $(D, N) = (2, 6)$, consider the simplified equation (3) for 70 variables, with a particular realization (the “alpha-realization”) of the pseudorandom numbers entering the equation. Other realizations give similar results.

Specifically, we take the following 6 pseudorandom numbers for the master momenta:

$$\widehat{p}_{\alpha\text{-realization}} = \left\{ \frac{53}{500},\ -\frac{9}{100},\ -\frac{441}{1000},\ \frac{217}{1000},\ \frac{371}{1000},\ \frac{19}{40} \right\}, \tag{5}$$

and the following 70 pseudorandom numbers for the master noise (splitting the matrices into real and imaginary parts):

3.3 Solutions of the simplified equation

$$\text{Re}\left[\widehat{\eta}^{\,1}_{\alpha\text{-realization}}\right] = \begin{pmatrix} -\frac{81}{125} & \frac{71}{1000} & -\frac{151}{500} & \frac{371}{500} & -\frac{83}{200} & \frac{491}{1000} \\ \frac{71}{1000} & -\frac{279}{1000} & -\frac{259}{500} & -\frac{13}{1000} & -\frac{493}{500} & \frac{449}{1000} \\ -\frac{151}{500} & -\frac{259}{500} & -\frac{413}{1000} & \frac{911}{1000} & \frac{203}{250} & \frac{299}{1000} \\ \frac{371}{500} & -\frac{13}{1000} & \frac{911}{1000} & \frac{671}{1000} & -\frac{417}{500} & -\frac{913}{1000} \\ -\frac{83}{200} & -\frac{493}{500} & \frac{203}{250} & -\frac{417}{500} & \frac{51}{125} & \frac{181}{250} \\ \frac{491}{1000} & \frac{449}{1000} & \frac{299}{1000} & -\frac{913}{1000} & \frac{181}{250} & \frac{261}{1000} \end{pmatrix}, \quad (6)$$

$$\text{Im}\left[\widehat{\eta}^{\,1}_{\alpha\text{-realization}}\right] = \begin{pmatrix} 0 & -\frac{441}{1000} & -\frac{17}{250} & -\frac{87}{1000} & -\frac{127}{200} & -\frac{199}{500} \\ \frac{441}{1000} & 0 & -\frac{177}{250} & -\frac{783}{1000} & -\frac{303}{500} & \frac{969}{1000} \\ \frac{17}{250} & \frac{177}{250} & 0 & -\frac{14}{25} & \frac{259}{1000} & -\frac{711}{1000} \\ \frac{87}{1000} & \frac{783}{1000} & \frac{14}{25} & 0 & \frac{43}{250} & \frac{1}{125} \\ \frac{127}{200} & \frac{303}{500} & -\frac{259}{1000} & -\frac{43}{250} & 0 & -\frac{491}{1000} \\ \frac{199}{500} & -\frac{969}{1000} & \frac{711}{1000} & -\frac{1}{125} & \frac{491}{1000} & 0 \end{pmatrix}, \quad (7)$$

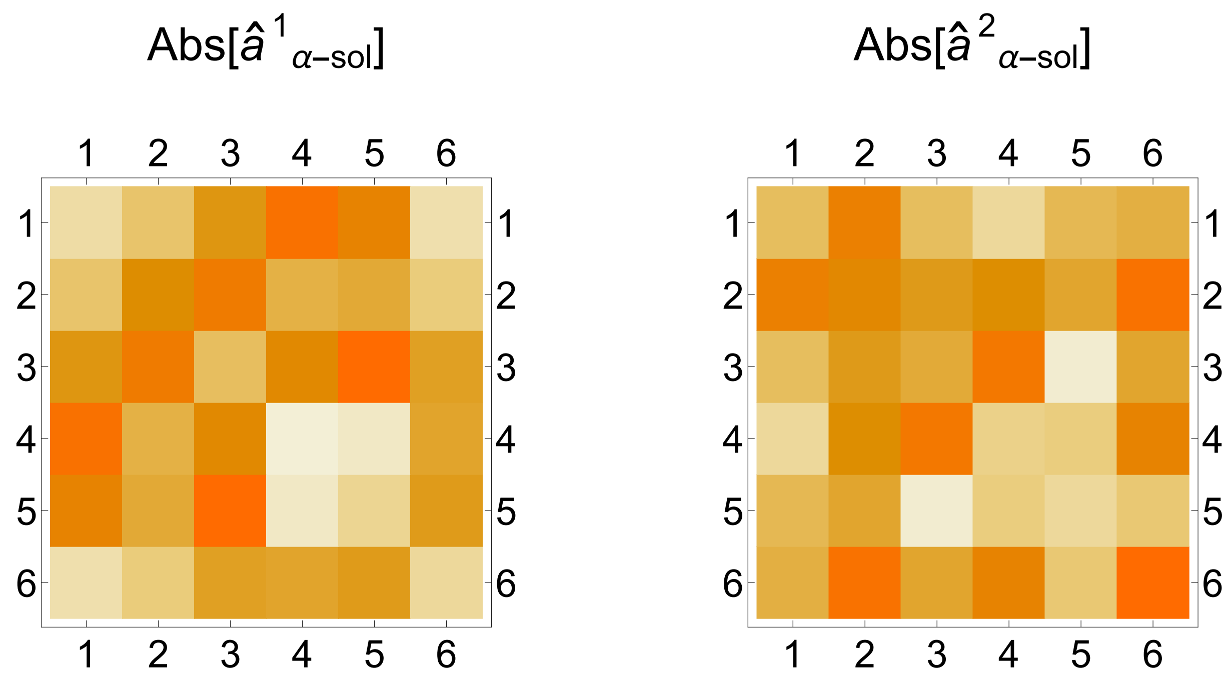
3.2 Solutions of the simplified equation

$$\text{Re}\left[\widehat{\eta}^{\,2}_{\alpha\text{-realization}}\right] = \begin{pmatrix} \frac{41}{200} & \frac{53}{1000} & -\frac{241}{250} & \frac{621}{1000} & \frac{3}{20} & -\frac{51}{200} \\ \frac{53}{1000} & -\frac{139}{500} & \frac{23}{200} & -\frac{557}{1000} & \frac{7}{100} & -\frac{137}{200} \\ -\frac{241}{250} & \frac{23}{200} & -\frac{31}{500} & -\frac{22}{125} & \frac{14}{25} & -\frac{31}{100} \\ \frac{621}{1000} & -\frac{557}{1000} & -\frac{22}{125} & \frac{289}{1000} & -\frac{227}{1000} & -\frac{103}{200} \\ \frac{3}{20} & \frac{7}{100} & \frac{14}{25} & -\frac{227}{1000} & \frac{17}{1000} & \frac{369}{1000} \\ -\frac{51}{200} & -\frac{137}{200} & -\frac{31}{100} & -\frac{103}{200} & \frac{369}{1000} & -\frac{171}{1000} \end{pmatrix}, \quad (8)$$

$$\text{Im}\left[\widehat{\eta}^{\,2}_{\alpha\text{-realization}}\right] = \begin{pmatrix} 0 & \frac{449}{500} & -\frac{31}{250} & \frac{233}{1000} & -\frac{413}{500} & -\frac{807}{1000} \\ -\frac{449}{500} & 0 & -\frac{56}{125} & \frac{7}{50} & -\frac{77}{200} & \frac{23}{500} \\ \frac{31}{250} & \frac{56}{125} & 0 & \frac{409}{500} & \frac{57}{250} & -\frac{689}{1000} \\ -\frac{233}{1000} & -\frac{7}{50} & -\frac{409}{500} & 0 & \frac{189}{500} & -\frac{953}{1000} \\ \frac{413}{500} & \frac{77}{200} & -\frac{57}{250} & -\frac{189}{500} & 0 & \frac{47}{200} \\ \frac{807}{1000} & -\frac{23}{500} & \frac{689}{1000} & \frac{953}{1000} & -\frac{47}{200} & 0 \end{pmatrix}, \quad (9)$$

3.2 Solutions of the simplified equation

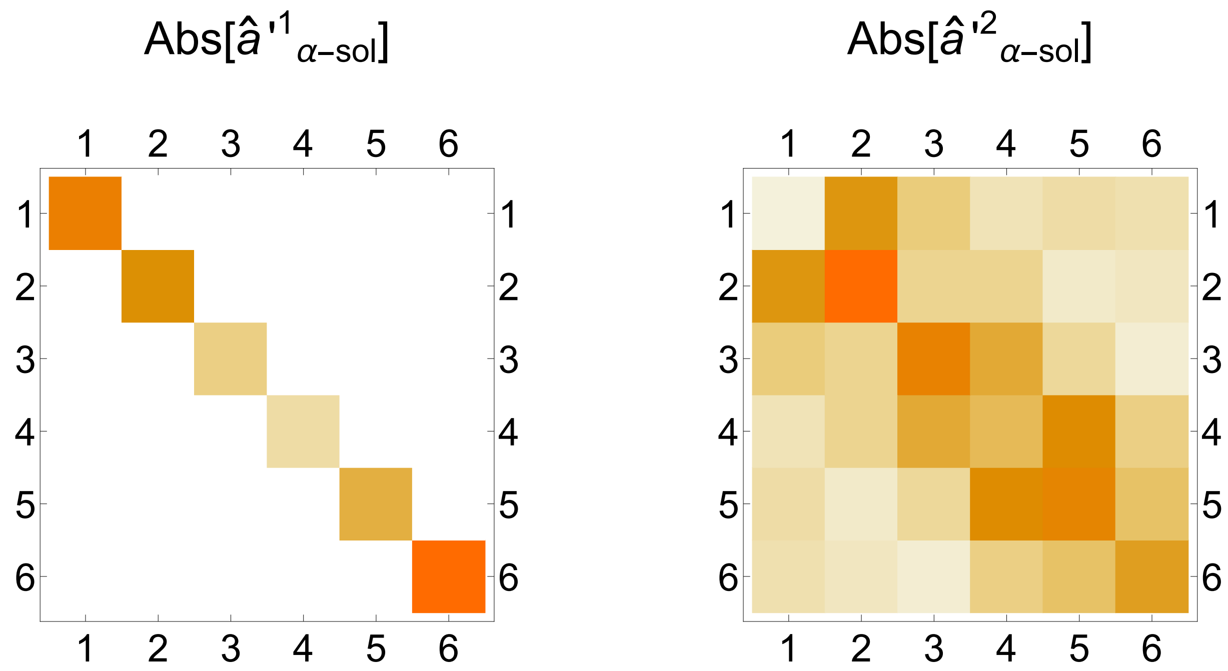
A particular solution [7] for the 70 variables of the simplified equation (3) with the alpha-realization of pseudorandom constants is given by $\widehat{a}^{\,1}_{\alpha\text{-sol}}$ and $\widehat{a}^{\,2}_{\alpha\text{-sol}}$. Consider the absolute values of the matrix entries:



$\Rightarrow$ no obvious band-diagonal structure.

3.2 Solutions of the simplified equation

Now, change the basis, in order to diagonalize and order the $\mu = 1$ matrix. This gives the matrices $\widehat{a}'^{\,1}_{\alpha\text{-sol}}$ and $\widehat{a}'^{\,2}_{\alpha\text{-sol}}$. Consider the absolute values of the matrix entries:



$\Rightarrow$ a diagonal/band-diagonal structure, a highly nontrivial result !

3.2 Solutions of the simplified equation

The values $(D, N) = (2, 6)$ are, of course, rather small. But ...

... scientists at Google Research, Zürich, now have obtained numerical solutions of the simplified equation (3) with $(D, N) = (10, 50)$ and these solutions apparently also display a diagonal/ band-diagonal structure [T. Fischbacher, private communication].

In short, work is in progress on solving and understanding the simplified algebraic equation ...

As mentioned before, the diagonal/band-diagonal structure of the master-field matrices allows for the extraction of a classical spacetime, but, first, we need to make sure that dynamical fermions do not spoil this structure.

3.3 Solutions of the full equation

We now look for solutions of the <u>full</u> bosonic master-field equation (1), with $F=1$ to include the dynamic fermions, but, first, with rather small values of D and N.

The Pfaffian is a K-th order polynomial, denoted symbolically by $P_K(A)$, with $K=(D-2)\,(N^2-1)$ according to (1c).

The basic structure of the algebraic equation (1) is then as follows:

$$P_1^{(\,\widehat{p}\,)}\,(\,\widehat{a}\,) \quad = \quad P_3\,(\,\widehat{a}\,)+F\,\frac{P_{K-1}\,(\,\widehat{a}\,)}{P_K\,(\,\widehat{a}\,)}+P_0^{(\,\widehat{\eta}\,)}\,(\,\widehat{a}\,)\;, \tag{10}$$

where the suffixes on P_1 and P_0 indicate their respective dependence on the master momenta $\widehat{p}_k$ and the master noise $\widehat{\eta}^{\,\mu}_{kl}$.

If we multiply (10) by $P_K\,(\,\widehat{a}\,)$, we get a polynomial equation of order $K+3$:

$$P_{K+1}^{(\,\widehat{p}\,)}\,(\,\widehat{a}\,) \quad = \quad P_{K+3}\,(\,\widehat{a}\,)+F\,P_{K-1}\,(\,\widehat{a}\,)+P_K^{(\,\widehat{\eta}\,)}\,(\,\widehat{a}\,)\;. \tag{11}$$

3.3 Solutions of the full equation

As a start, we have considered the case

$$\{D,\, N,\, F\} = \{3,\, 3,\, 1\}\,, \tag{12}$$

for which the model still has a supersymmetry invariance and the eight generators T^I are proportional to the 3×3 Gell-Mann matrices λ^I. The bosonic matrices are expanded as $A_\mu = A_I^\mu\, T^I$, with real coefficients A_I^μ.

Remarkably, there is now an explicit result [9] for the Pfaffian,

$$\begin{aligned}\mathcal{P}_{3,\,3}[A] \;=\; & -\frac{3}{4}\,\text{Tr}\Big(\big[A^\mu,\, A^\nu\big]\,\big\{A^\rho,\, A^\sigma\big\}\Big)\,\text{Tr}\Big(\big[A^\mu,\, A^\nu\big]\,\big\{A^\rho,\, A^\sigma\big\}\Big)\\ & +\frac{6}{5}\,\text{Tr}\Big(A^\mu\,\big[A^\nu,\, A^\rho\big]\Big)\text{Tr}\Big(A^\mu\,\big[\{A^\nu,\, A^\sigma\},\, \{A^\rho,\, A^\sigma\}\big]\Big),\end{aligned} \tag{13}$$

which corresponds to a real homogenous eighth-order polynomial in the bosonic coefficients A_I^μ.

3.3 Solutions of the full equation

Taking an explicit realization of the random constants, we have established the existence of several solutions of the full bosonic master-field equation (1) for the case $(D, N) = (3, 3)$.

Moreover, there is a suggested diagonal/band-diagonal structure, but the value $N = 3$ is too small for definitive statements. Further details can be found in Ref. [8].

3.3 Solutions of the full equation

The result for the case (12) was obtained by a direct algebraic calculation. Further progress may be obtained by an indirect numerical approach.

The idea (emphasized to me by Jun Nishimura) is to use the fact that the square of the Pfaffian of the skew-symmetric matrix $\mathcal{M} = \mathcal{M}(\widehat{a})$ equals its determinant,

$$\Big[\mathrm{Pf}(\mathcal{M})\Big]^2 = \det \mathcal{M}\,, \tag{14a}$$

so that we can write the variational term in the algebraic equation (1) as a trace,

$$\frac{1}{\mathrm{Pf}(\mathcal{M})}\,\delta\,\mathrm{Pf}(\mathcal{M}) = \frac{1}{2}\,\mathrm{Tr}\Big[\mathcal{M}^{-1}\delta\,\mathcal{M}\Big]\,, \tag{14b}$$

and this trace can be evaluated numerically (as used in Ref. [10] and earlier papers). This work is still in progress, see the next slide.

3.3 Solutions of the full equation

Table 1: Numerical solutions of the full ($F = 1$) bosonic master-field equation (1). The number of variables N_{var} is given by (4) and the equation is a polynomial of order $N_{\text{poly}} = K + 3$, with K given by (1c).

	N_{var}	N_{poly}	status
$(D,\, N) = (3,\, 3)$	24	11	done ($\sim 1/2$ hr) [a]
$(D,\, N) = (10,\, 3)$	80	67	done (~ 76 hrs)
$(D,\, N) = (10,\, 4)$	150 [b]	123	work in progress (?)

[a] previous algebraic results reproduced

[b] complex variables in the solution, as the Pfaffian $\mathcal{P}(\widehat{a})$ is complex

4. Conclusions

It is conceivable that a **new physics phase** gives rise to classical spacetime, gravity, and matter, as described by our current theories (General Relativity and the Standard Model).

For an explicit calculation, we have considered the **IIB matrix model**, which has been proposed as a nonperturbative formulation of type-IIB superstring theory (M-theory).

The crucial insight is that the emergent classical spacetime may reside in the **large-N master field** $\widehat{A}^{\mu}$ of the IIB matrix model.

We have now started to **solve** the full bosonic master-field equation of the IIB matrix model: first results are in, but the road ahead is long and arduous ...

5. References

[1] N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, “A large-N reduced model as superstring,” Nucl. Phys. B **498**, 467 (1997), arXiv:hep-th/9612115.

[2] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, A. Tsuchiya, and T. Tada, “IIB matrix model,” Prog. Theor. Phys. Suppl. **134**, 47 (1999), arXiv:hep-th/9908038.

[3] E. Witten, “The $1/N$ expansion in atomic and particle physics,” in G. ’t Hooft et. al (eds.), *Recent Developments in Gauge Theories*, Cargese 1979 (Plenum Press, New York, 1980).

[4] J. Greensite and M.B. Halpern, “Quenched master fields,” Nucl. Phys. B **211**, 343 (1983).

[5] F.R. Klinkhamer, “IIB matrix model: Emergent spacetime from the master field,” Prog. Theor. Exp. Phys. **2021**, 013B04 (2021), arXiv:2007.08485.

[6] F.R. Klinkhamer, “M-theory and the birth of the Universe,” Acta Phys. Pol. B **52**, 1007 (2021), arXiv:2102.11202.

5. References

[7] F.R. Klinkhamer, “A first look at the bosonic master-field equation of the IIB matrix model,” to appear in Int. J. Mod. Phys. D, arXiv:2105.05831.

[8] F.R. Klinkhamer, “Solutions of the bosonic master-field equation from a supersymmetric matrix model,” arXiv:2106.07632.

[9] W. Krauth, H. Nicolai, and M. Staudacher, “Monte Carlo approach to M-theory,” Phys. Lett. B **431**, 31 (1998), arXiv:hep-th/9803117.

[10] K.N. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura, T. Okubo, and S. K. Papadoudis, “Complex Langevin analysis of the spontaneous breaking of 10D rotational symmetry in the Euclidean IKKT matrix model,” JHEP **06**, 069 (2020), arXiv:2002.07410.

A. Background (short version)

The algebraic equation of interest arises from the IKKT matrix model [1]. That model is also known as the IIB matrix model [2], as the matrix model reproduces the basic structure of the light-cone string field theory of type-IIB superstrings.

The IIB matrix model has a finite number of $N \times N$ traceless Hermitian matrices: ten bosonic matrices A^μ and eight fermionic (Majorana–Weyl) matrices Ψ_α.

The partition function Z of the IIB matrix model is defined by the following "path" integral:

$$Z = \int dA\, d\Psi\; e^{-\,S(A,\,\Psi)} = \int dA\, d\Psi\; e^{-S_{\text{bos}}(A)\,-\,S_{\text{ferm}}(\Psi,\,A)}, \quad \text{(A.1)}$$

where the bosonic action $S_{\text{bos}}(A)$ is quartic in A and the fermionic action $S_{\text{ferm}}(\Psi,\, A)$ is quadratic in Ψ and linear in A, i.e., $S_{\text{ferm}} = \overline{\Psi}\,\mathcal{M}(A)\,\Psi$.

A. Background (short version)

The fermionic matrices Ψ can be integrated out exactly (Gaussian integrals) and give the Pfaffian Pf$[\mathcal{M}(A)] \equiv \mathcal{P}(A)$:

$$Z \;=\; \int dA\, \mathcal{P}(A)\, e^{-S_{\text{bos}}(A)} \equiv \int dA\, e^{-S_{\text{eff}}(A)}\,. \tag{A.2}$$

For the bosonic observable

$$w^{\mu_1 \cdots \mu_m} = \text{Tr}\left(A^{\mu_1} \cdots A^{\mu_m}\right), \tag{A.3}$$

and arbitrary strings thereof, the expectation values are defined by the same integral as in (A.2):

$$\begin{aligned} &\left\langle w^{\mu_1 \cdots \mu_m}\, w^{\nu_1 \cdots \nu_n} \,\cdots\, w^{\omega_1 \cdots \omega_z} \right\rangle \\ &= \frac{1}{Z} \int dA \left(w^{\mu_1 \cdots \mu_m}\, w^{\nu_1 \cdots \nu_n} \,\cdots\, w^{\omega_1 \cdots \omega_z}\right) e^{-S_{\text{eff}}}\,. \end{aligned} \tag{A.4}$$

A. Background (short version)

Now, the IIB matrix model just gives **numbers**, Z and the expectation values $\langle w\, w\, \cdots \rangle$, while the matrices A^μ and Ψ_α in the “path” integral are merely integration variables.

Moreover, there is no obvious small dimensionless parameter to motivate a saddle-point approximation.

The **conceptual** question arises: $\boxed{\text{where is the classical spacetime?}}$

Recently, we have suggested to revisit an old idea, the large-N master field of Witten [3, 4], for a possible origin of classical spacetime in the context of the IIB matrix model [5].

A. Background (short version)

According to Witten [3], the large-N factorization of the expectation values (A.4) implies that the path integrals are saturated by a single configuration, the so-called **master field** $\widehat{A}^{\,\mu}$.

To leading order in N, the expectation values are then given by

$$\langle w^{\mu_1\,\cdots\,\mu_m}\, w^{\nu_1\,\cdots\,\nu_n}\,\ldots\, w^{\omega_1\,\cdots\,\omega_z}\rangle \stackrel{N}{=} \widehat{w}^{\,\mu_1\,\cdots\,\mu_m}\,\widehat{w}^{\,\nu_1\,\cdots\,\nu_n}\,\ldots\,\widehat{w}^{\,\omega_1\,\cdots\,\omega_z}\,, \tag{A.5a}$$

$$\widehat{w}^{\,\mu_1\,\cdots\,\mu_m} \equiv \mathrm{Tr}\left(\widehat{A}^{\,\mu_1}\,\cdots\,\widehat{A}^{\,\mu_m}\right). \tag{A.5b}$$

Hence, we do not have to perform the path integrals on the right-hand side of (A.4): we just need ten traceless Hermitian matrices $\widehat{A}^{\,\mu}$ to get *all* these expectation values from the simple procedure of replacing each $A^{\,\mu}$ in the observables by the corresponding $\widehat{A}^{\,\mu}$.

A. Background (short version)

Now, the meaning of the suggestion at the top of slide 5 is clear:

classical spacetime may reside in the bosonic master-field matrices $\widehat{A}^{\,\mu}$ of the IIB matrix model.

The heuristics of this idea has been discussed in Sec. 4.4 of Ref. [6].

Assuming that the matrices $\widehat{A}^{\,\mu}$ of the IIB-matrix-model master field are known and that they are approximately band-diagonal (as suggested by the numerical results of Ref. [10] and references therein), it is possible [5] to extract a discrete set of spacetime points $\{\widehat{x}_k^{\,\mu}\}$ and an interpolating (inverse) metric $g^{\mu\nu}(x)$.

It has been established that, in principle, it is possible to get, from appropriate distributions of the extracted spacetime points $\{\widehat{x}_k^{\,\mu}\}$, the metrics of the Minkowski and the spatially flat Robertson–Walker spacetimes. See the recent review [6] for further discussion.

GEV-75

Congratulations

to

Grisha Volovik

on his (upcoming) 75th birthday!

GEV-75

For some 15 years, I have had the privilege to collaborate with Grisha Volovik, a great physicist.

Our main work has been on the Cosmological Constant Problem and goes under the name of q-theory.

Just a sketch of how this q is supposed to operate the Universe:

$\boxed{\text{Big Bang}} \longrightarrow \boxed{\text{Now}}$

Big Bang	Now
$T \sim E_{\text{Planck}}$	$T \sim 3\,K$
$q(A)$	$q(A)$
A is a microscopic variable	q is a macroscopic variable, which self-adjusts
$\Lambda_{\text{eff}} \sim (E_{\text{Planck}})^4$	to give: $\Lambda_{\text{eff}} \sim 0$

The present talk is about the physics of the Big Bang $\rightarrow$ slide 1