Dynamic pairing orders and odd frequency superconductivity

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Volovik 75’th

Sept 6 2021

Acknowledge : ERC, KAW

Happy Anniversary!

What is odd f (Berezinskii) pairing

- Ubiquitous pairing state requires time component
- Odd f state in Josephson junction
- Dynamic order

- Composite pairing – Cooper + boson
- No Go Thm for Eliashberg framework
- Present in Majorana systems
- Topological

J. Linder, AVB,
Odd F SC
RMP (2019)

Symmetry of the Pairing Correlator

The superconducting anomalous pairing function is :
. Due to Fermi statistics

$$\Delta_{\alpha\beta}(k,\tau) = \langle T c_\alpha(\tau) c_\beta(0) \rangle$$

$$\Delta_{\alpha\beta}(\mathbf{k}) = -\Delta_{\beta\alpha}(-\mathbf{k})$$

spin-singlet *s*-wave or spin-triplet *p*-wave

orbital

spin

$$\Delta_{\alpha\beta}(\mathbf{k}) = \Delta_0 e^{i\varphi} \eta(\mathbf{k}) \chi_{\alpha\beta}$$

The pair correlation can also be odd in time/frequency: [1]

Odd-frequency spin-singlet *p*-wave or spin-triplet *s*-wave

$$\Delta_{\alpha\beta}(\mathbf{k},\omega) = -\Delta_{\beta\alpha}(-\mathbf{k},-\omega)$$

[1]: Berezinskii, JETP Lett. 20, 287 (1974)

[1]: Berezinskii, JETP Lett. 20, 287 (1974)

$$\mathrm{PT}[\vec{\Delta}(\mathbf{r},\tau)] = \vec{\Delta}(\mathrm{r},\tau)$$

$$PT = -1$$

$$S = 1$$

$P = -1, T = +1$	$P = +1, T = -1$
$p-wave$	$s-wave$
time even	*time* odd triplet
BCS triplet	odd-frequency pairing

Why odd frequency state is interesting:

- Why 1974- Superfluid He3
- Correlated states: Satisfies no double occupancy constraints

$$\left\langle \psi_{\uparrow}^{+}\left(r_1 t\right)\psi_{\downarrow}^{+}\left(r_1 t\right)\right\rangle = \sum_{k,\omega} \Delta(k,\omega) = 0$$

- Superconducting spintronics
- Dynamic order in driven and dissipative systems
- Topology of Odd f – Zero ABS- Majorana state

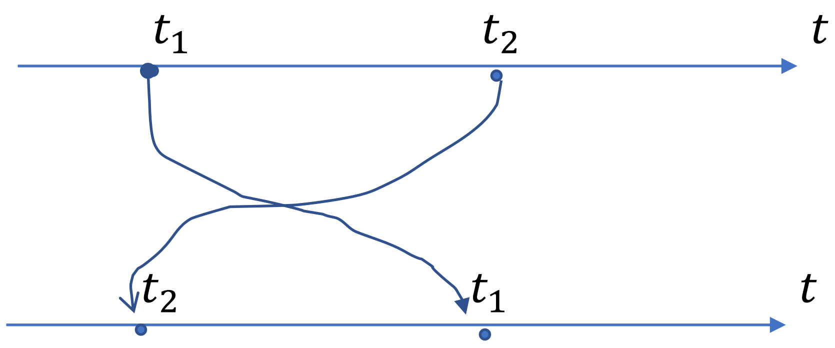
Prelim I: define

- $S\,\Delta_{\alpha\beta\;ab}(r,r'|\tau,\tau')S^{-1} = \Delta_{\beta\alpha\;ab}(r,r'|\tau,\tau')$
- $P^*\,\Delta_{\alpha\beta\;ab}(r,r'|\tau,\tau')P^{*-1} = \Delta_{\alpha\beta\;ab}(r',r|\tau,\tau')$ - coordinate permut
- $T^*\,\Delta_{\alpha\beta\;ab}(r,r'|\tau,\tau')T^{*-1} = \Delta_{\alpha\;\beta\;ab}(r',r|\tau',\tau)$- time permut
- $O\,\Delta_{\alpha\beta\;ab}(r,r'|\tau,\tau')O^{-1} = \Delta_{\alpha\;\beta\;ba}(r',r|\tau',\tau)$

- Rewrite as S – spin indices permutation, P*- space permutation, T* - time permutation, O- orbital index permutation
- SP*OT* Δ = - Δ

$$\Delta_{\alpha\beta}(\mathbf{k},\omega) = -\Delta_{\beta\alpha}(-\mathbf{k},-\omega)$$

Prelim II
Not a time reversal !



Ber state does not have to break time reversal

Classification of Pairing states

- Pairing state: $P_O^{ab}(r,\tau) = \langle \mathcal{T} O^a(r,\tau) O^b(0,0) \rangle \neq 0$

  O^a: fermions, bosons, spins, …

Classification of Pairing states

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- Statistics demands product of SP*T* = -1 for fermion, (O = +1)

$$P^{ab}(r,\tau) = -P^{ba}(-r,-\tau)$$

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$$P^{ab}(r,\tau) = -P^{ba}(-r,-\tau)$$

$$P^{ab}(r,r'|\tau,\tau') = < T O_a(r,\tau) O_b(r',\tau') >$$

$$P^{ab}(r,r'|\tau,\tau') = (\pm 1)^{B,F} P^{ba}\left(r',r \;\middle|\; \tau',\tau\right)$$

- Allow same classification for Bosons and Fermions
- no translational invariance in time, space
- connect to time crystals and odd f in driven driven systems

8 fold classification

SP*OT* = -1

Dynamic order

8 classes of SC (BCS vs Berezinskii)

-	-	-	-	-	-	-	-	-	
S	-	-	-	-	+	+	+	+	
P*	+	+	-	-	-	-	+	+	
O	+	-	+	-	+	-	+	-	
T*	+	-	-	+	+	-	-	+	
	BCS S wave				BCS P wave				

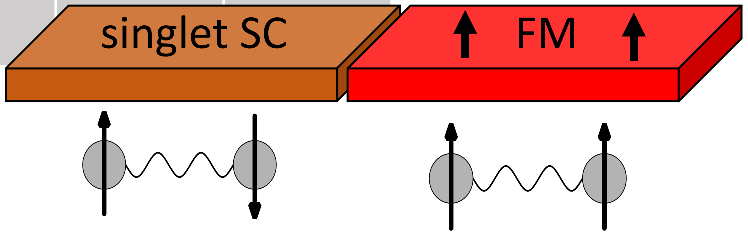
SP*OT* = -1, S^2=P*^2=T*^2=O^2 = +1 -> 2^4 possibilities, ½ consistent with (-1) -> 8 fold possible pairings

Realizations of Ber pairing

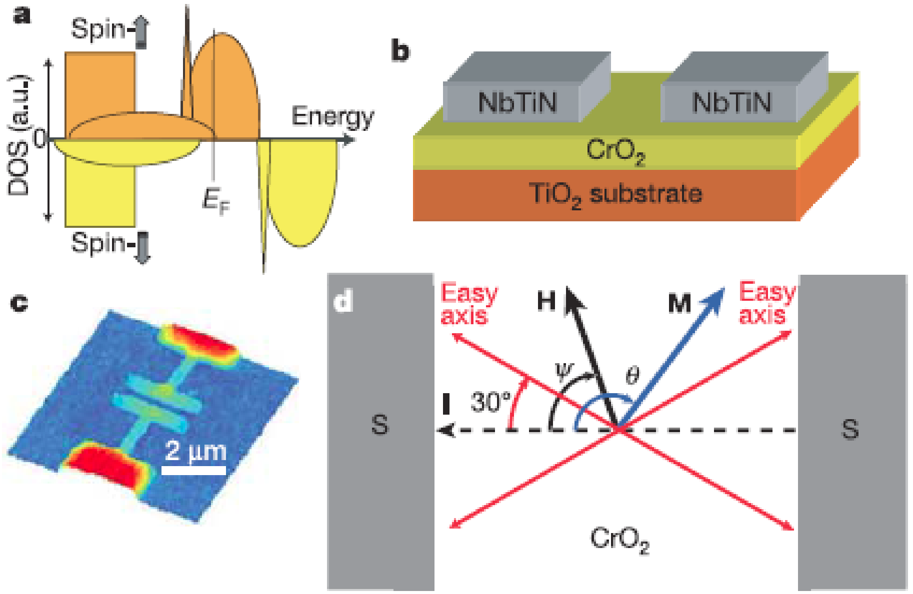
Design principles for Ber state in heterostructures

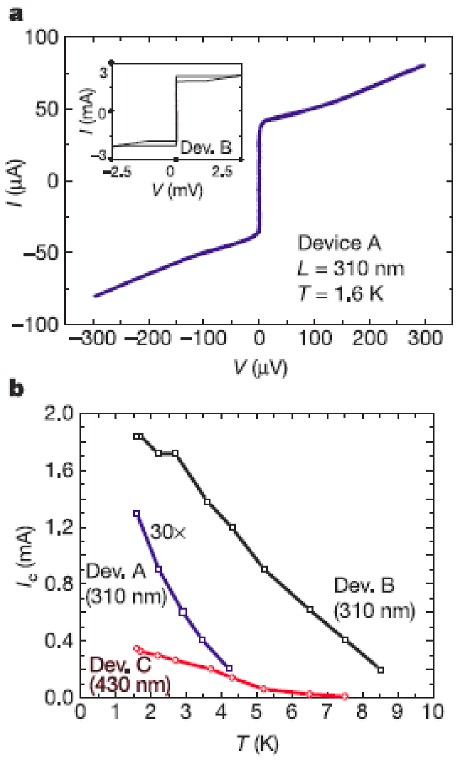
-	-	-	-	-	-	-	-	-	
S	-	-	-	-	+	+	+	+	
P*	+	+	-	-	-	-	+	+	
O	+	-	+	-	+	-	+	-	
T*	+	-	-	+	+	-	-	+	
	BCS S wave				BCS P wave				

Scattering in SC/FM heterojunction



Experimental Evidence: Sc/FM





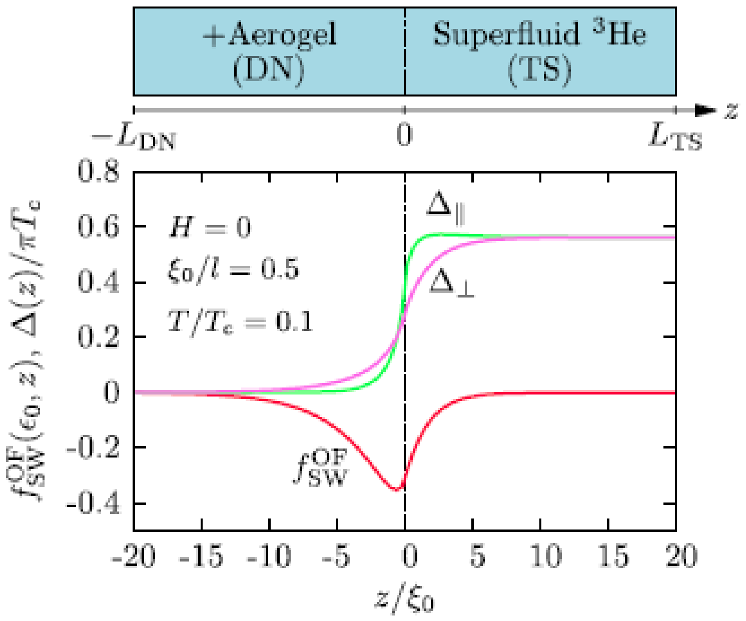
R. S. Keizer, S. T. B. Goennenwein, T. M. Klapwijk, G. Miao, G. Xiao, and A. Gupta, Nature (London) 439, 825 (2006).

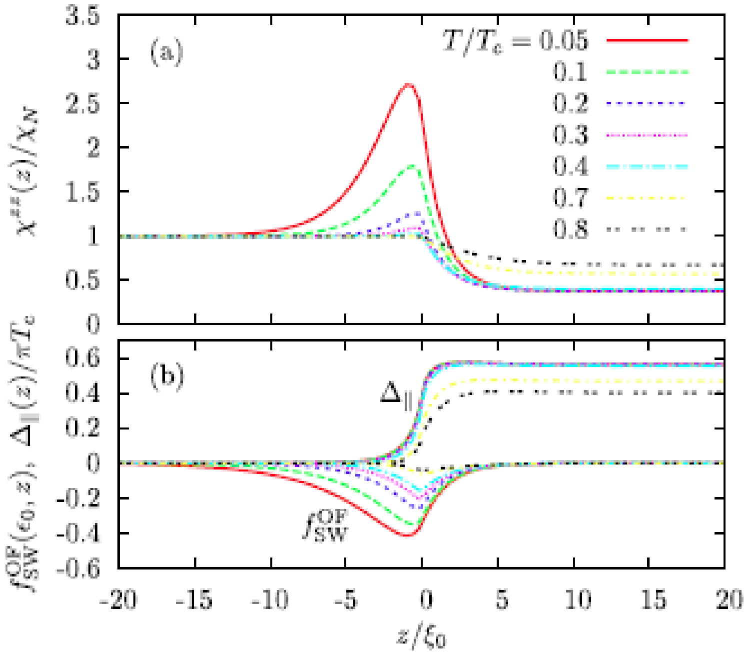
Related: 3He Superfluid example

PRL **110,** 175301 (2013) PHYSICAL REVIEW LETTERS week ending 26 APRIL 2013

Magnetic Response of Odd-Frequency *s*-Wave Cooper Pairs in a Superfluid Proximity System

S. Higashitani, H. Takeuchi, S. Matsuo, Y. Nagato, and K. Nagai

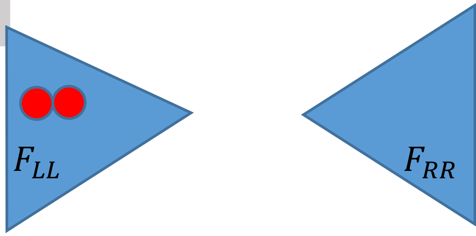




DESIGN PRINCIPLES FOR BER STATE IN HETEROSTRUCTURES

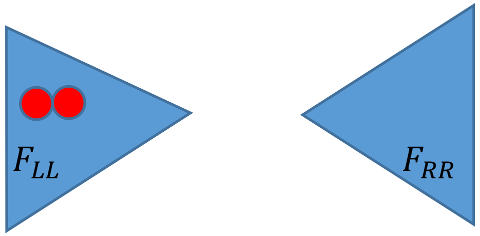
L,R

-	-	-	-	-	-	-	-	-	
S	-	-	-	-	+	+	+	+	
P*	+	+	-	-	-	-	+	+	
O	+	-	+	-	+	-	+	-	
T*	+	-	-	+	+	-	-	+	
	BCS S wave				BCS P wave				



In JJ, all odd in tunneling T processes, L-R odd

Odd F in Josephson effect



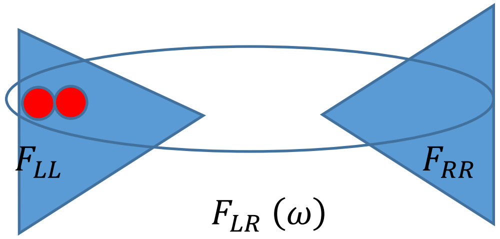
Left, right lead distort due
to pair tunneling

In Josephson description Cooper pairs are not broken:

only LL and RR pairs are allowed: S = -1, P* = +1, T * = +1, O = +1,

Odd F in Josephson effect

$$H = H_0 + T \sum_{spin,k,k'} c_L^*(k) c_R(k') + h.c.$$



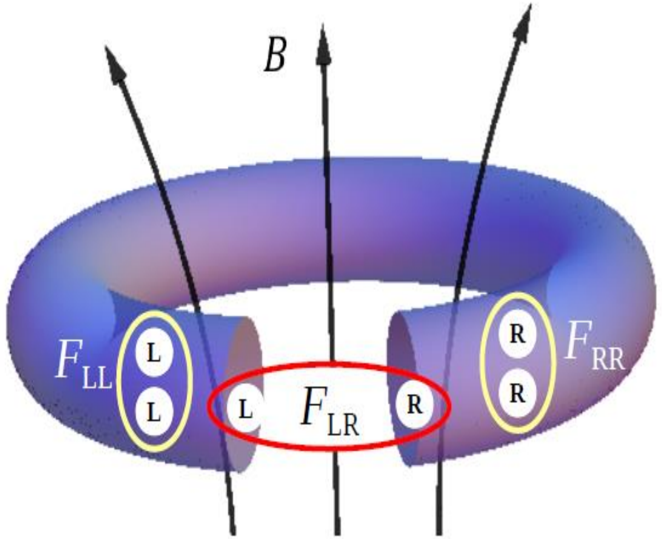
Odd frequency anomalous pair in JJ = pair fractionalization!
Pairs are separated across

$$F_{LR}(\omega) = \omega\,[\Delta_L - \Delta_R]/D_L\,D_R \sim \omega\,|\Delta|\sin[(\phi_L - \phi_R)/2$$

Odd f, odd L-R: S = -1, P* = +1, T* = -1, O = -1,

SP*OT* = -1

Observables



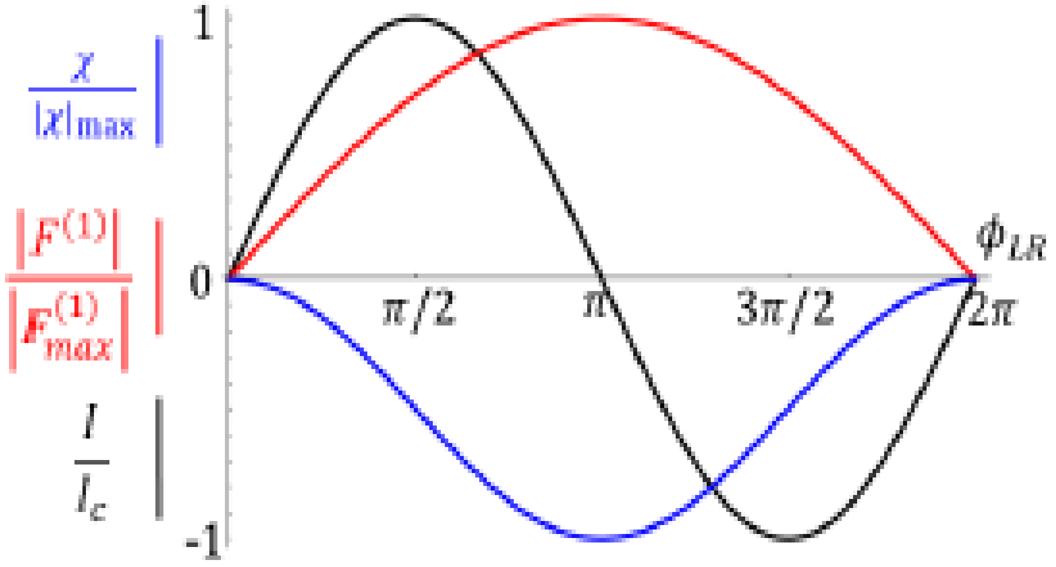


FIG. 2. Dependence on the Josephson phase, ϕ_{LR}, of: the odd-frequency amplitude, Eq (8), (red); Josephson current, Eq (9), (black); and interlead spin-susceptibility, Eq (11), (blue). For the spin-susceptibility, only the contribution due to the anomalous terms, $F^{(1)}$, which are odd in ω, was used.

Finite T = 0

Spin susceptibility T=0

In s-wave SC!

$$\chi_{LR} = < S_L S_R > |_{\omega \to 0} \sim \mathrm{T}^2 \Delta \sin(2\phi_{LR}) \sim I_J^2$$

$$F_{LR}(t) = \pi^2 N_0^2 \mathrm{T} \Delta e^{\frac{\{i\phi_L + i\phi_R\}}{2}} \sin(2\phi_{LR})\, e^{i\Delta t}$$

1804.07244,
RMP v91, 045005(2019)

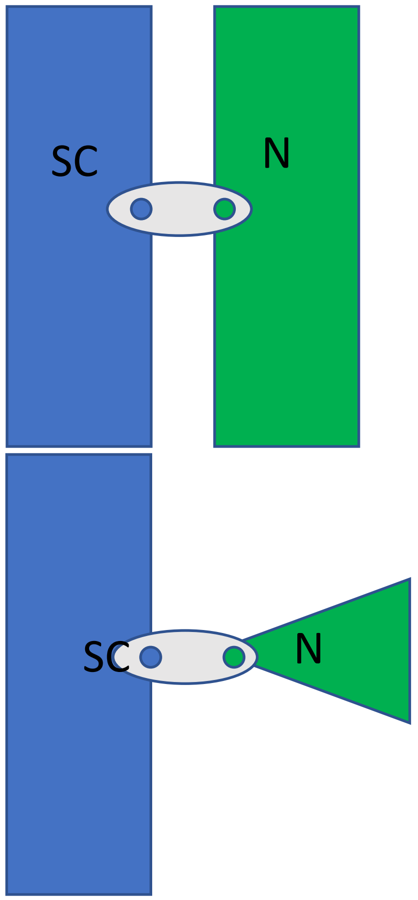
S-N junction induction of odd f SC

Take F_RR = 0

On site $F_{LR}(\omega) = i\omega\,\Delta_L/(\omega^2 + \Delta_L^2)$

Berezinskii state in N/S junction!

normal STM tip + SC

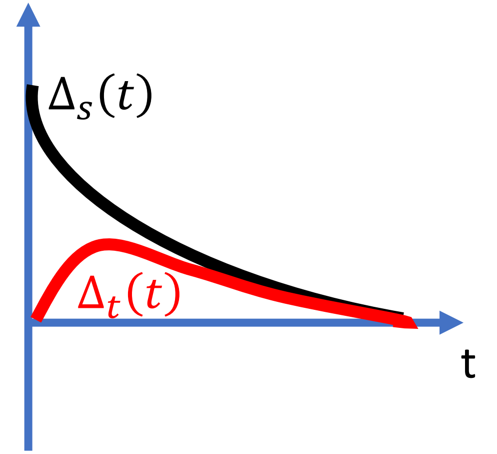


Dynamic induction of Ber pairing

Consider spin decay $$c_\sigma(t) = c_\sigma \exp -\frac{t}{\tau_\sigma}, \sigma = \uparrow,\downarrow$$

Pair fields: $$\Delta_s(t) = \langle c_\uparrow(t) c_\downarrow \rangle - \uparrow\leftrightarrow\downarrow = \left(e^{-\frac{t}{\tau_\uparrow}} + e^{-\frac{t}{\tau_\downarrow}}\right) \langle c_\uparrow c_\downarrow \rangle \sim ch(t(\tau_\uparrow - \tau_\downarrow))$$

$$\Delta_t(t) = \langle c_\uparrow(t) c_\downarrow \rangle + \uparrow\leftrightarrow\downarrow = \left(e^{-\frac{t}{\tau_\uparrow}} - e^{-\frac{t}{\tau_\downarrow}}\right) \langle c_\uparrow c_\downarrow \rangle \sim sh(t(\tau_\uparrow - \tau_\downarrow))$$



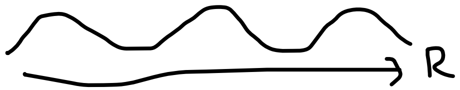
Ber is universally present in dynamics:

- Scattering – heterostructures FM/SC
- Floquet
- Pump
- Non Eq steady state - JJ
- Dissipation

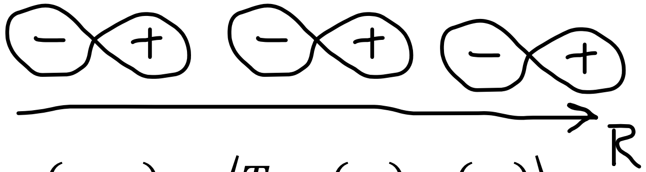
S. Bandyopadhay, preprint (2021)
J. Linder, AVB, RPM (2019)

Odd F Berezinskii as a dynamic order

Order/state	**Static orders**
	$R = r_1 + r_2,$ $r = r_1 - r_2$
X tal	X tal
SC	SC:P-wave, d-wave



$$\rho(r_1 r_2) = \langle c^*(r_1) c(r_2) \rangle = \rho(R)$$
$$= A\cos(QR)$$

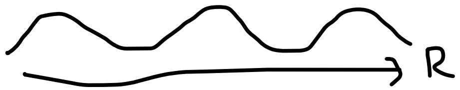


$$\Delta_{\alpha\beta}(r_1 r_2) = \left\langle T c_\alpha\,(r_1) c_\beta(r_2) \right\rangle =$$
$$\Delta_{\alpha\beta}(r) = A\sin(Qr)$$

Odd F Berezinskii as a dynamic order

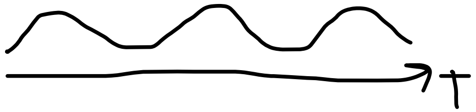
Order/state	**Static orders**	**Dynamic orders**
	$R = r_1 + r_2,$ $r = r_1 - r_2$	$T = t_1 + t_2$ $t = t_1 - t_2$
X tal	X tal	Time Xtal
SC	SC:P-wave, d-wave	Berezinskii odd f pairing

X tal (static):



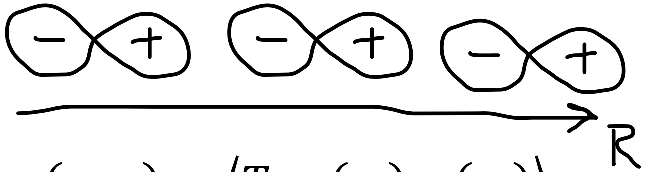
$$\rho(r_1 r_2) = \langle c^*(r_1) c(r_2) \rangle = \rho(R) = A\cos(QR)$$

Time Xtal (dynamic):



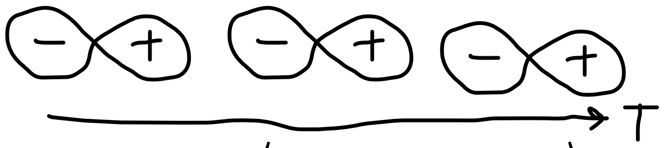
$$\rho(t_1, t_2) = \langle c^*(t_1) c(t_2) \rangle = \rho(T) = A\cos(\Omega T)$$

SC:P-wave, d-wave (static):



$$\Delta_{\alpha\beta}(r_1 r_2) = \left\langle T c_\alpha(r_1) c_\beta(r_2) \right\rangle = \Delta_{\alpha\beta}(r) = A\sin(Qr)$$

Berezinskii odd f pairing (dynamic):



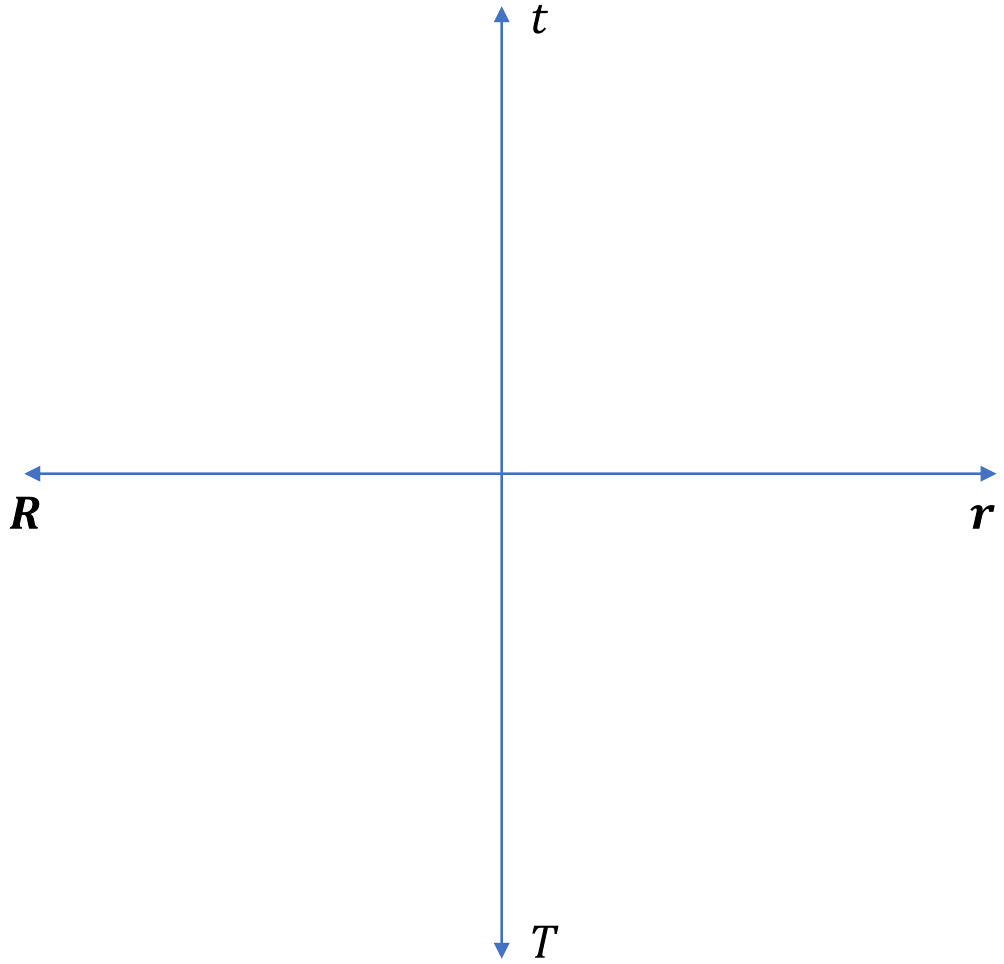
$$\Delta_{\alpha\beta}(t_1 t_2) = \left\langle T c_\alpha(t_1) c_\beta(t_2) \right\rangle = \Delta_{\alpha\beta}(t) = A\sin(Qt)$$

Dynamic Ber orders <cc>

$$\langle c(\boldsymbol{r}_1, t_1) c(\boldsymbol{r}_2, t_2) \rangle = F(T, R | t, r)$$

$$\boldsymbol{R} = \boldsymbol{r}_1 + \boldsymbol{r}_2, \qquad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$$

$$T = t_1 + t_2, \qquad t = t_1 - t_2$$



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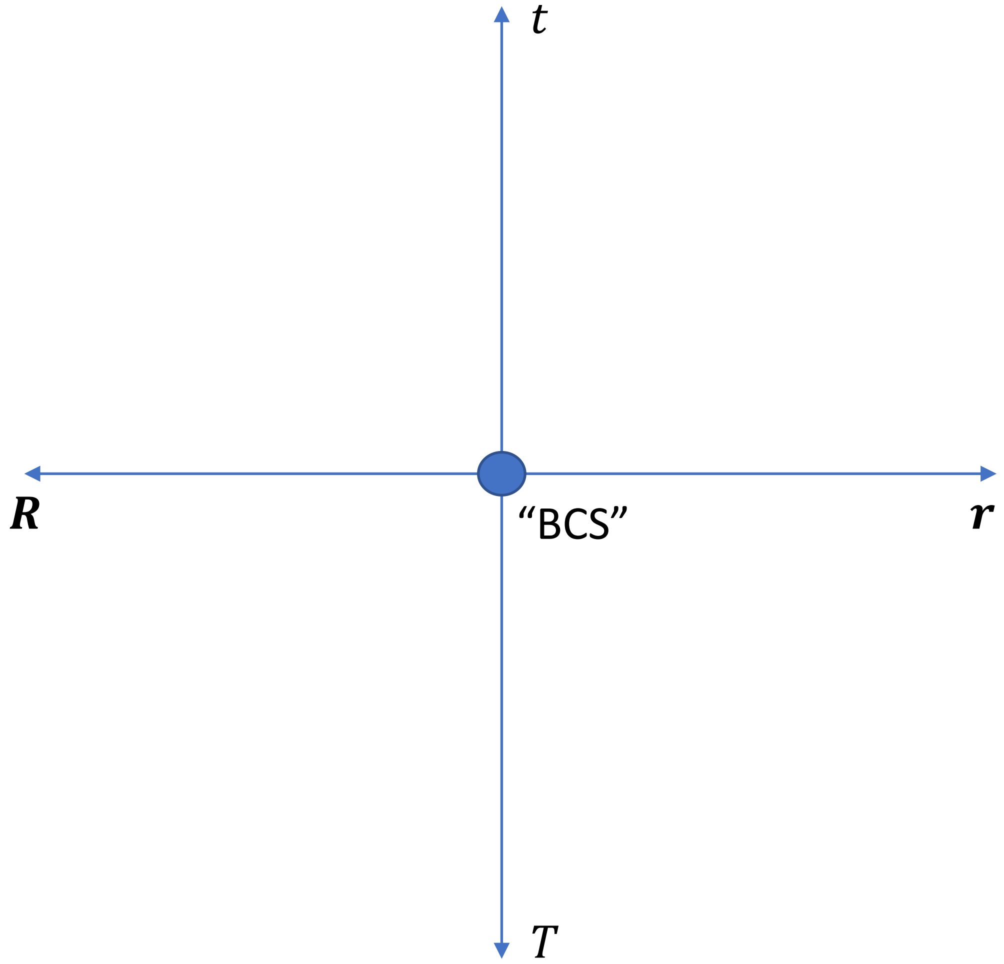
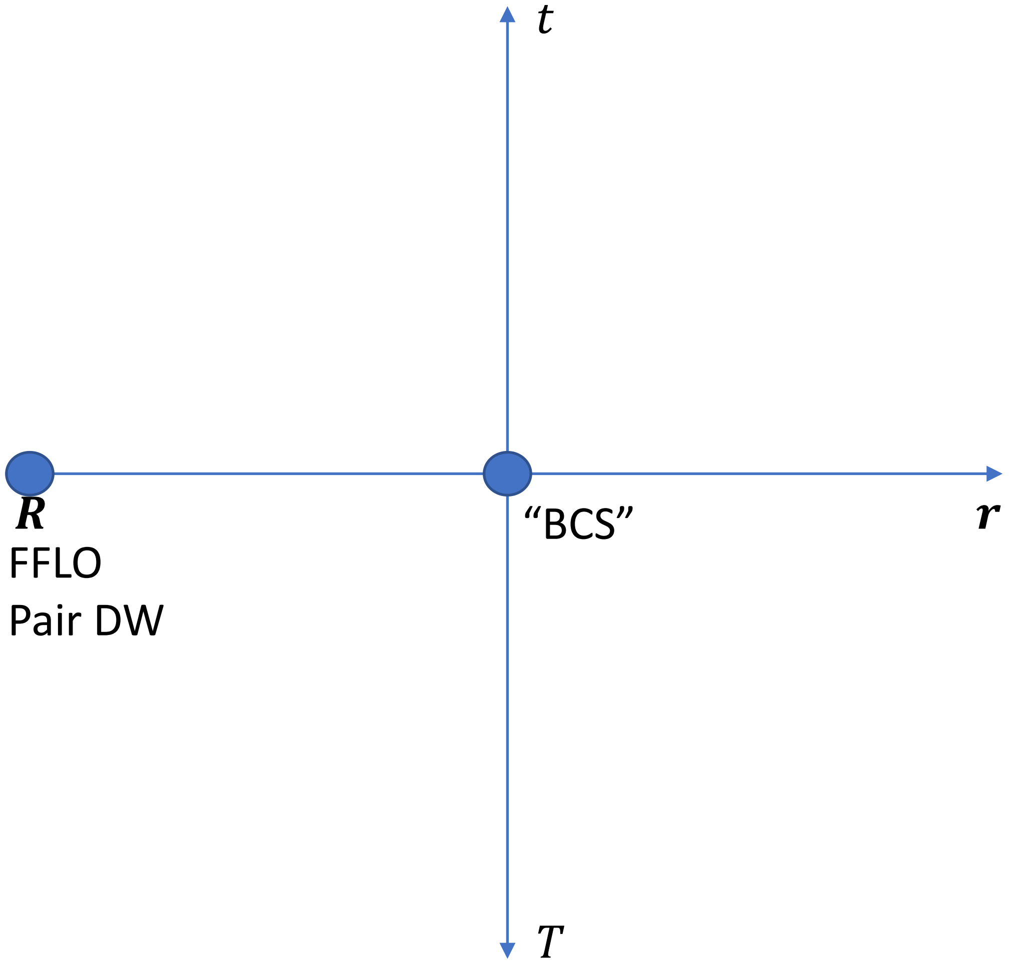


Dynamic Ber orders
<cc>

$$\langle c(\boldsymbol{r}_1, t_1) c(\boldsymbol{r}_2, t_2) \rangle = F(T, R | t, r)$$

$$\boldsymbol{R} = \boldsymbol{r}_1 + \boldsymbol{r}_2, \qquad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$$

$$T = t_1 + t_2, \qquad t = t_1 - t_2$$



Dynamic Ber orders <cc>

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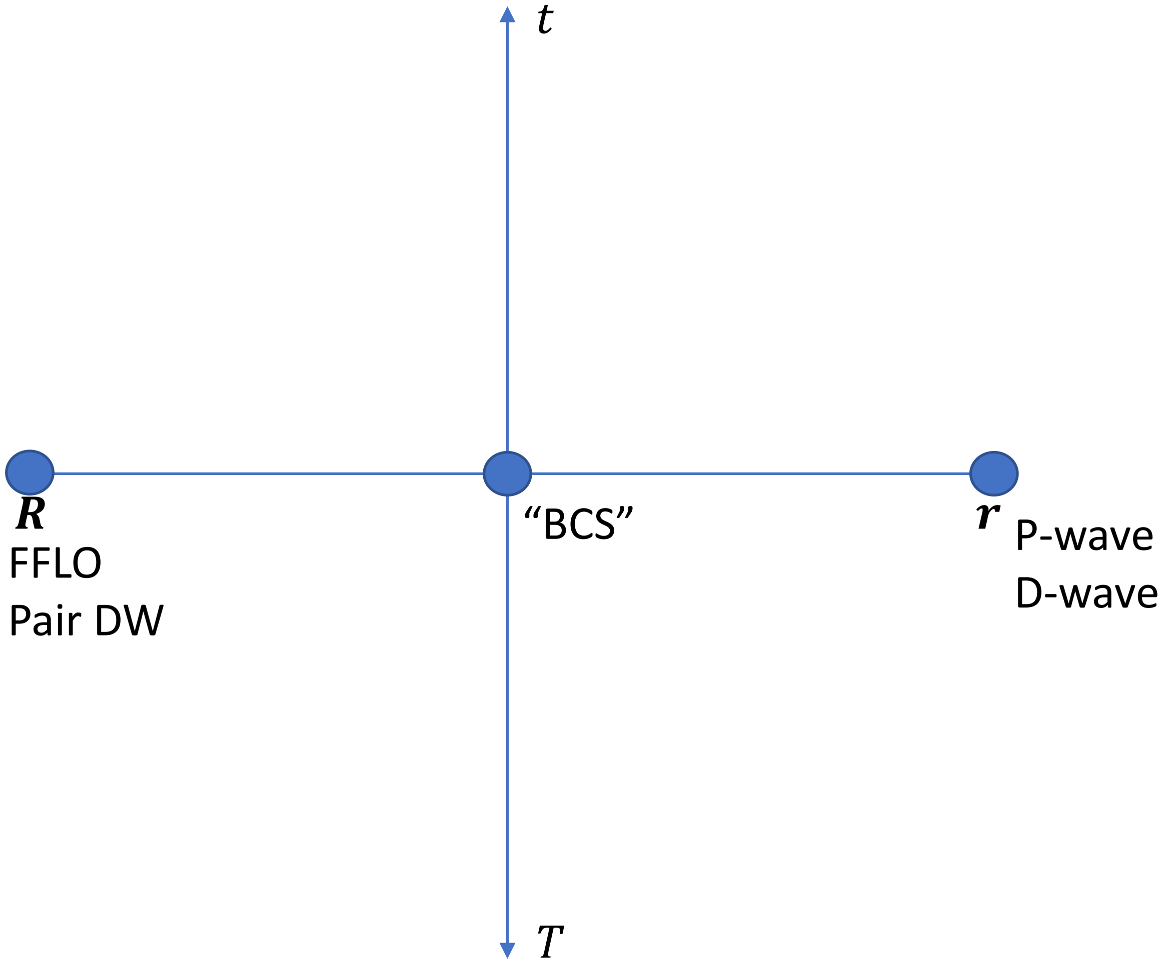

Dynamic Ber orders <cc>

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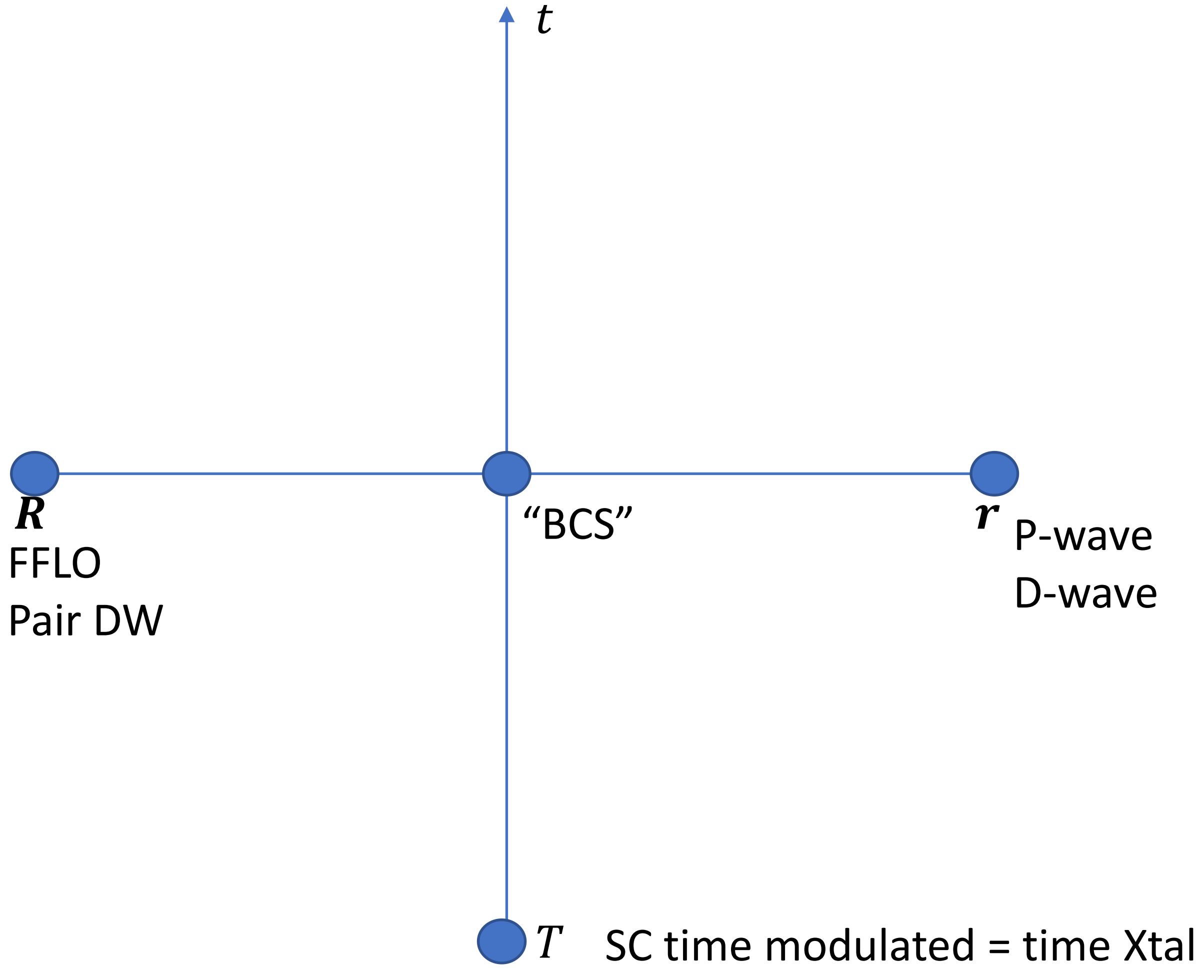

Dynamic Ber orders <cc>

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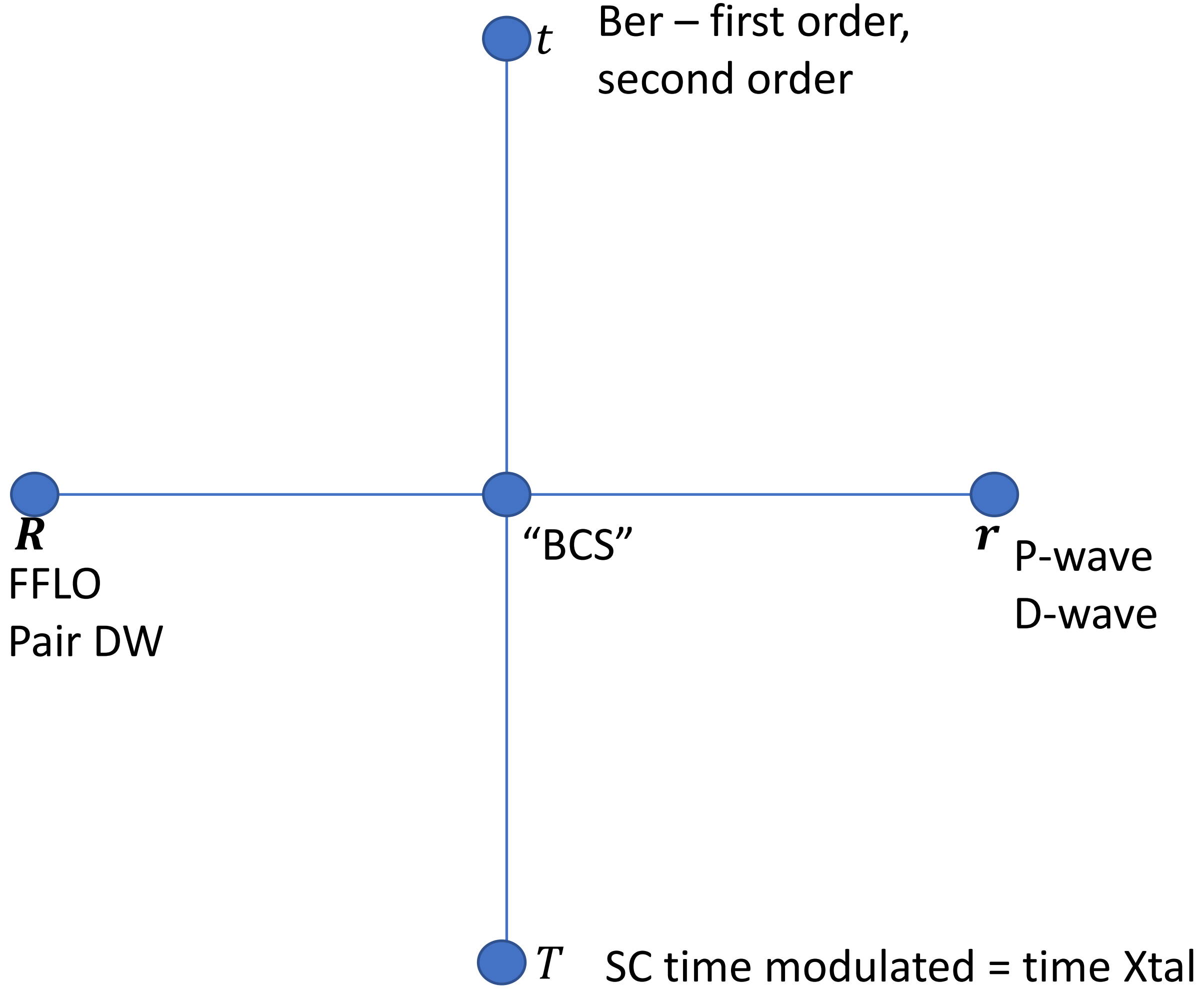

Dynamic Ber orders <cc>

$$\langle c(\boldsymbol{r}_1, t_1) c(\boldsymbol{r}_2, t_2)\rangle = F(T, R|t, r)$$

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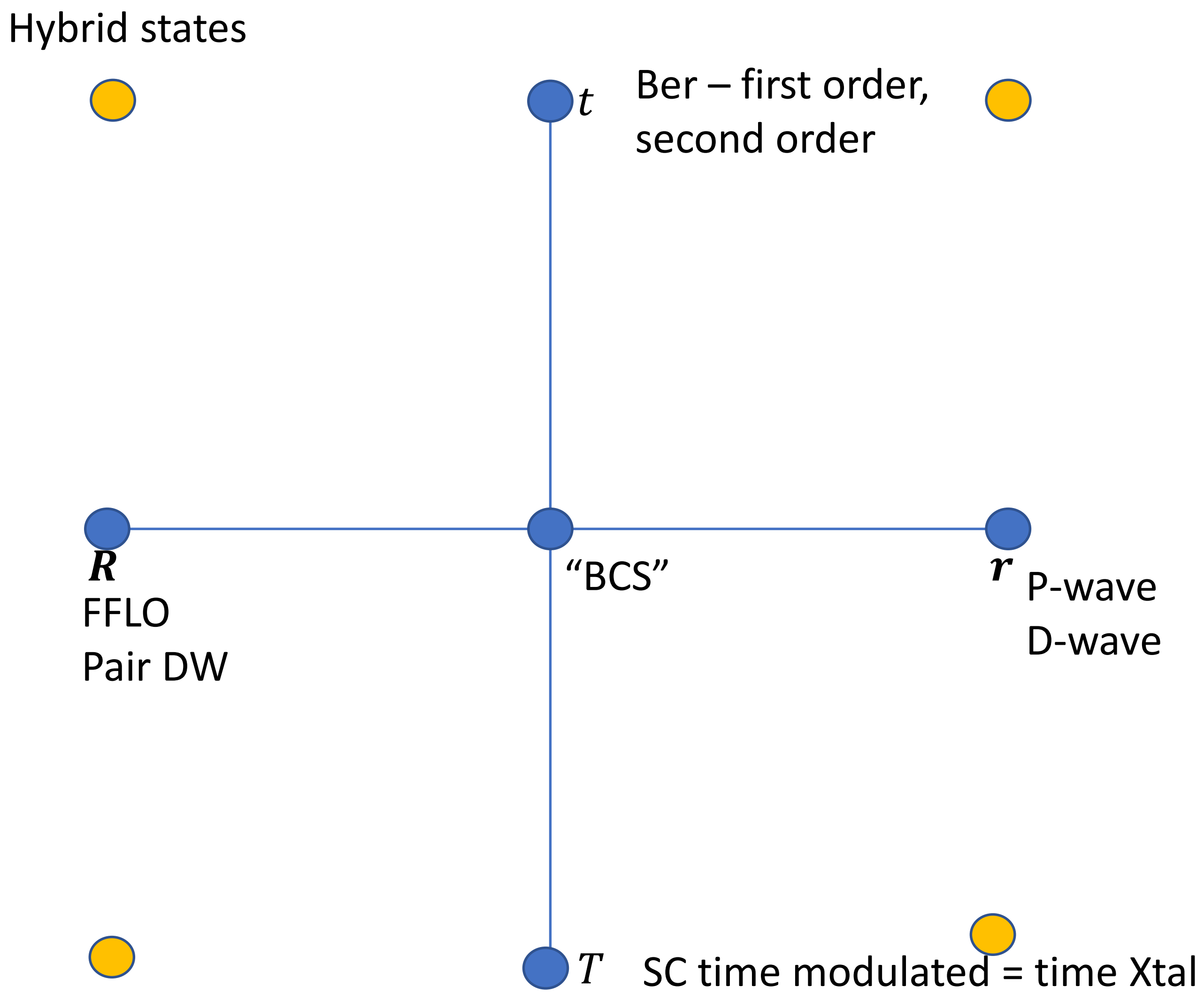

Dynamic Ber orders <cc>

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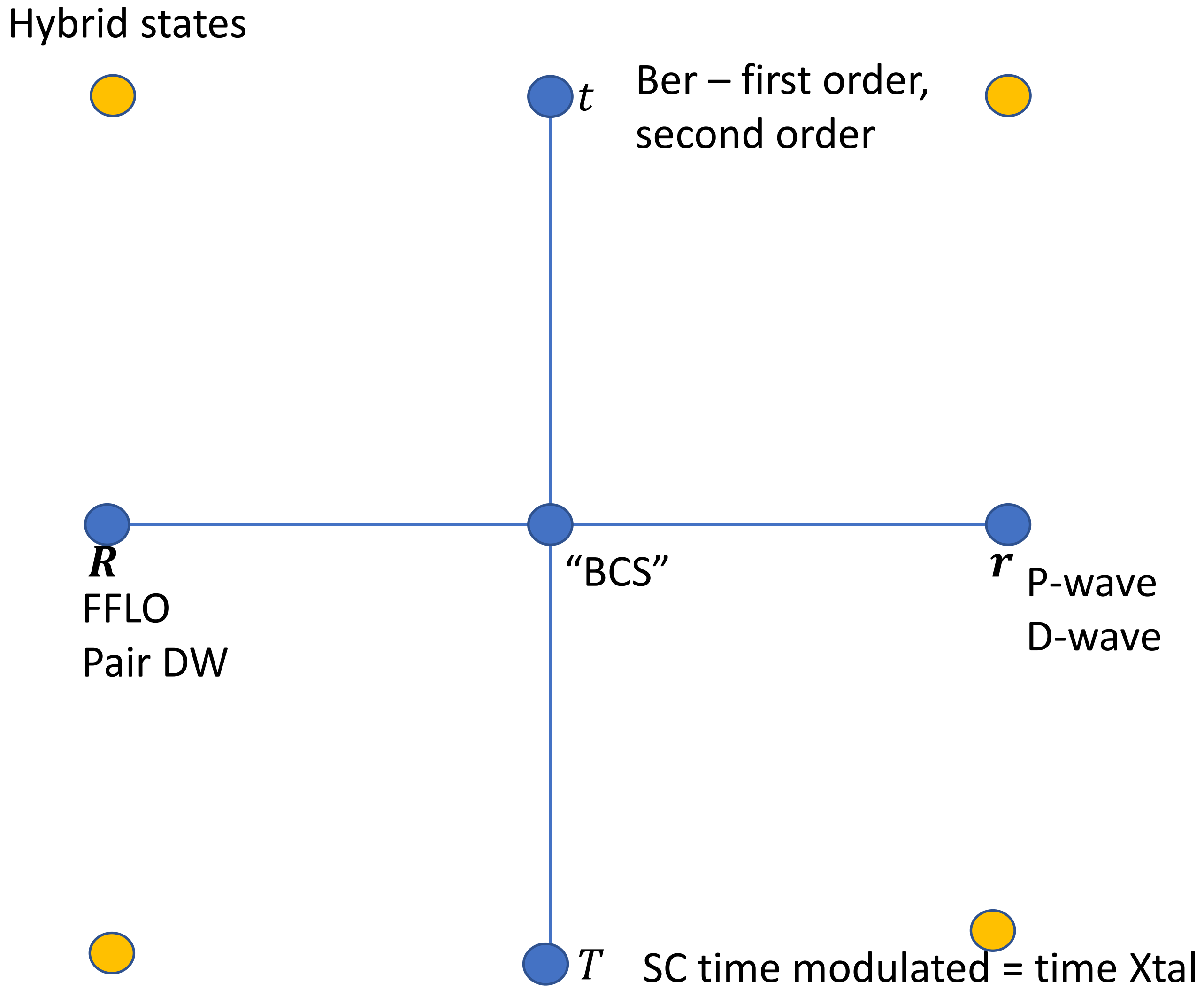

What’s next?

Extension to PH states

Better understanding of dynamics

Topology of Ber – K theory

Dynamic Ber orders
<c*c>- ph channel

$$\langle c^*(\boldsymbol{r}_1, t_1) c(\boldsymbol{r}_2, t_2)\rangle = F(T, R | t, r)$$

$$\boldsymbol{R} = \boldsymbol{r}_1 + \boldsymbol{r}_2, \qquad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$$

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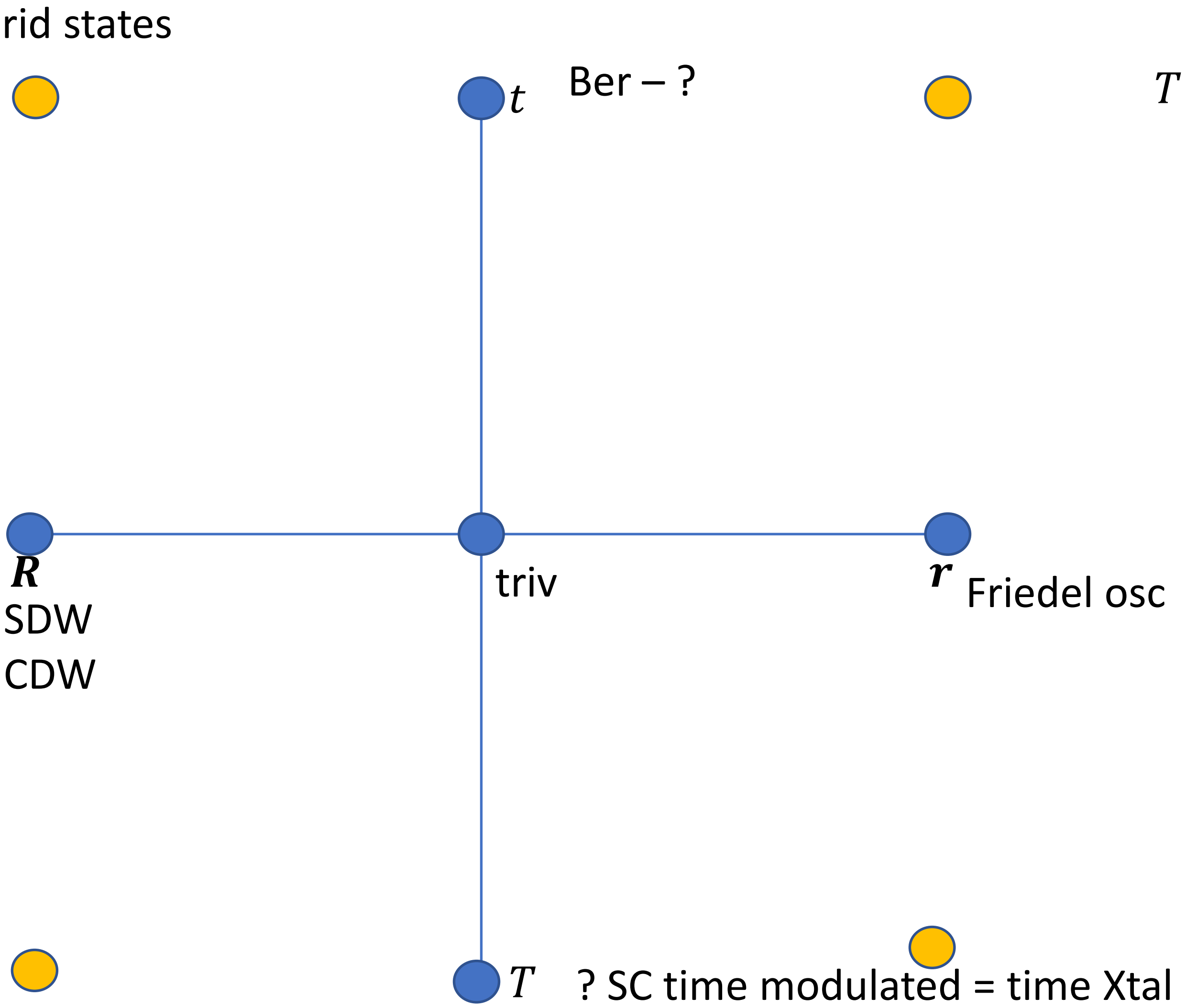
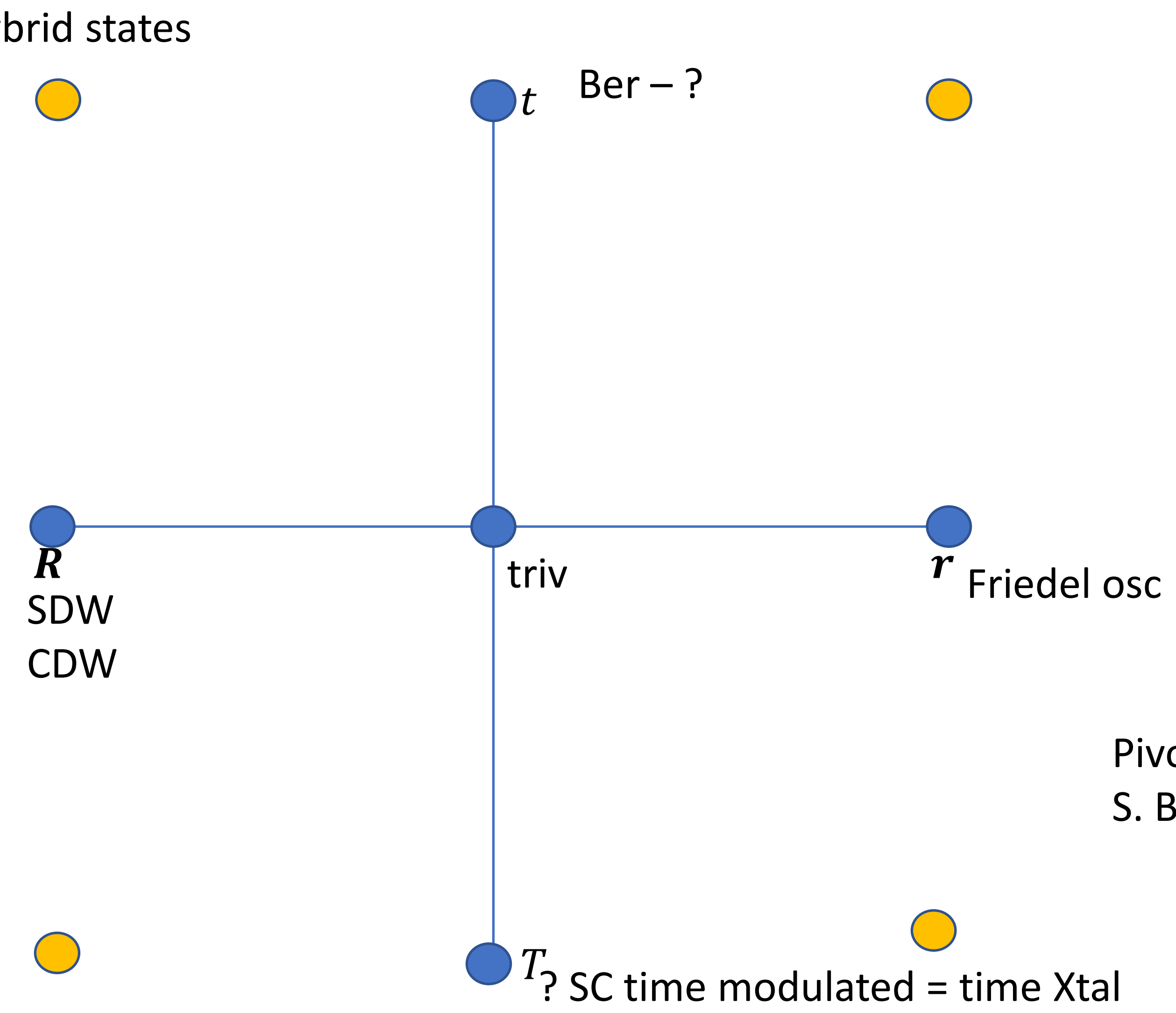


Dynamic Ber orders
<c*c>- ph channel

Ber classification
C*SPOT



Pivovarov, Nayak, PRB64, 035107(2001)
S. Bandyopadhay, AB (2021)

Odd f superconductivity players

- P. Coleman  V. Emery  Y. Fominov 
- A. Tsvelik  S. Kivelson 
- Y. Tanaka  T. Kirkpatrick  D.Belitz 
- M. Eschrig  K. Efetov 
- J.Linder  Annica Black Schaffer 

and more ….

H. Dahal APS
J. Bonca JSI
D. Mozyrsky LANL
E. Abrahams, Rutgers/UCLA
JR Schrieffer U Florida
D. Scalapino UCSB

A. Black-Schaffer, UU
S. Pershoguba, C. Triola, M. Geilhufe, Y. Kedem Nordita
G. Fernando UCONN
S. Nakosai RIKEN
Z. Huang, LANL
P. Wolfle KIT

Conclusion

- Odd F SC- dynamic order
- 8 fold possible pairing states
- Examples – in Josephson junctions, in heterostructures and in Dirac materials
- Future directions:
- Ber pairing in dynamics
- Ber in pairing of quark-gluon plasma
- Odd f density waves
- Topology of Ber

Happy Anniversary!